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Problem 1 (MCQs)

a) C: satisfies G = G$_f$
b) A: True
c) B: False
d) A: True
e) B: False
f) B: False
g) A: True
h) B: False

Problem 2 (Edge on MST)

Algorithm: We are given a weighted graph G = (V, E) with positive weights, c_i for all $i \in E$. To determine whether an input edge $e = (u, v) \in E$ with weight c_e is part of the MST, we follow the following steps.

We run a Depth-First Search starting at vertex u and traverse through edges of the graph G which have a weight lower than c_e.

-   If this DFS traversal reaches the vertex v, then we determine that the input edge e is not part of the MST due to the cycle property of MST. The cycle property states that for any cycle C in the graph, if the weight of an edge e of C is larger than the individual weights of all other edges of C, then this edge cannot belong to MST. Since during DFS, we traversed through only those edges that had weights strictly smaller than the weight of edge e to get from vertex u to v, if we were to add the edge e = (u, v) to MST, it would violate this property since c_e is larger than all other individual weights in the u -> v cycle.

- If this DFS traversal does not reach vertex v, then we know edge e is not the edge with the maximum weight in any of the cycles it is a part of. And hence, it can be added to the MST.

Runtime: Since the algorithm basically runs a DFS to see if it can reach a certain vertex, the runtime is $O(|E| + |V|)$. Therefore, the algorithm runs in linear time as expected.

Correctness: The cycle property states that for any cycle C in the graph, if the weight of an edge e of C is larger than the individual weights of all other edges of C, then this edge cannot belong to MST. Since during DFS, we traversed through only those edges that had weights strictly smaller than the weight of edge e to get from vertex u to v, the only way a path exists between vertex u and v is if weight of edge e was the highest in one of the cycles it was a part of. And that would make edge e inadmissible to the MST. On the contrary, the only reason a DFS would not find a path between u and v in such a case is if edge e does not have the maximum weight in any of the cycles it is a part of. And hence, it can be a part of some MST of G.